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SELF-SIMILAR SOLUTIONS TO THE PROBLEM OF THE MOTION OF A SPHERICAL PISTON IN A HEAT-CONDUCTING MEDIUM WITH CONSERVATION OF THE ENERGY OF A POINT EXPLOSION

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In this paper we study the problem of the motion of a spherical piston with fixed heat removal on it along a heat-conducting medium with a distributed density, in which there initially occurred a point explosion which released a finite energy E_0. We study the case when the heat removal is compensated by the work performed by the piston, i.e., the total energy of the medium remains constant and equal to the released energy E_0.

Analysis of the numerically found self-similar solutions revealed the following features.

For solutions which have the same total energy, as the velocity of the piston and the rate of heat removal on it increase, the mass velocity or propagation of the forward wave front, the difference between the velocity of the forward front of the perturbations and the velocity of the shock wave following it, and the relative fraction of the thermal energy all decrease.

As E_0 increases, first of all, the behavior indicated above intensifies and, second, interesting features are observed for two limiting problems – a pure explosion [1, 2] and maximum heat removal: the percentage of the kinetic energy of the explosion in the problem without the piston (pure explosion) drops and the percentage of the kinetic energy of the explosion in the problem with maximum heat removal (the temperature at the piston equals zero) increases.

We write the system of gas-dynamics equations in the Lagrangian mass coordinate system [3] in the form

$$\frac{\partial r}{\partial t}=v,\ \frac{\partial r}{\partial m}=\frac{1}{\rho r^2},\ \frac{\partial v}{\partial t}=-r^2\frac{\partial p}{\partial m},\ \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial (r^2 v)}{\partial m},$$
$$\frac{\partial \varepsilon}{\partial t}=-p\frac{\partial (r^2 v)}{\partial m}-\frac{\partial (r^2 w)}{\partial m},\ W=-r^2\rho\varkappa\frac{\partial T}{\partial m}, \tag{1}$$
$$\varepsilon=\frac{RT}{\gamma-1},\ p=R\rho T,\ \varkappa=aT^{5/2},\ \gamma=\frac{5}{3}.$$

Here r is the radius, m is the Lagrangian mass variable, t is the time, v is the velocity, ρ is the density, p is the pressure, ε is the internal energy, T is the temperature, W is the heat flux, and $\varkappa$ is the coefficient of thermal conductivity, characteristic for a high-temperature hydrogen plasma.

Dimensional analysis [4] shows that the problem of an instantaneous point explosion followed by the motion of a spherical piston has a self-similar solution when the following hold:

boundary conditions on the piston (m = 0)

$$v(0,\ t)=v_0t^{-1/4},\ W(0,\ t)=-v(0,\ t)p(0,\ t); \tag{2}$$

boundary conditions on the forward front of the perturbation wave $m_N(t)$

$$\rho(m_N,\ t)=\rho_0 m_N^{-7/2},\ v(m_N,\ t)=T(m_N,\ t)=W(m_N,\ t)=0; \tag{3}$$

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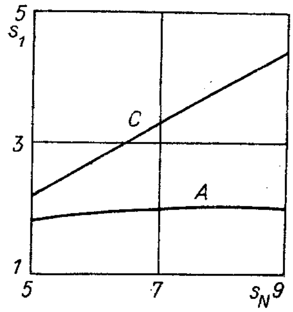

Fig. 1

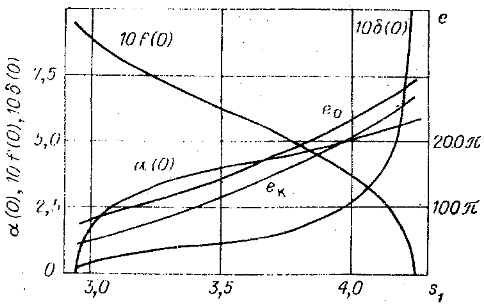

Fig. 2

conditions of conservation of the total energy of the medium

$$4\pi \int\limits_0^{m_N} \left(\frac{T}{\gamma - 1} + \frac{v^2}{2}\right) dm = E_0. \tag{4}$$

The boundary-value problem (1)-(4) has a self-similar solution, for which the following formulas for the transformation to one independent dimensionless variable s are valid:

$$\begin{gathered} m = t^{1/2}R^{7/16}\rho_0^{1/4}a^{-1/8}s,\ r(m,\,t) = t^{3/4}R^{21/32}\rho_0^{1/24}a^{-3/16}\lambda(s), \\ v(m,\,t) = t^{-1/4}R^{21/32}\rho_0^{1/24}a^{-3/16}\alpha(s),\ \rho(m,\,t) = t^{-7/4}R^{-49/32}\rho_0^{1/8}a^{7/16}\delta(s), \\ T(m,\,t) = t^{-1/2}R^{5/16}\rho_0^{1/12}a^{-3/8}f(s),\ W(m,\,t) = t^{-5/2}R^{7/16}\rho_0^{1/4}a^{-1/8}\varphi(s). \end{gathered} \tag{5}$$

Using these transformation formulas, we write the boundary-value problem for the dimensionless functions depending on s:

system of equations

$$\begin{gathered} \frac{d\lambda}{ds} = \frac{1}{\delta\lambda^2},\ \frac{s}{2}\frac{d\lambda}{ds} - \frac{3}{4}\lambda = -\alpha,\ \frac{s}{2}\frac{d\alpha}{ds} + \frac{\alpha}{4} = \lambda^2\frac{d(\delta f)}{ds}, \\ \frac{s}{2}\frac{d\delta}{ds} + \frac{7}{5}\delta = \delta^2\frac{d(\lambda^2\alpha)}{ds},\ \frac{0{,}5}{\gamma - 1}\frac{d(sf)}{ds} = \delta f\frac{d(\lambda^2\alpha)}{ds} + \frac{d(\lambda^2\varphi)}{ds}, \\ \varphi = -\lambda^2\delta f^{5/2}\frac{df}{ds},\ \gamma = \frac{5}{3}; \end{gathered} \tag{6}$$

condition on the piston (s = 0)

$$\alpha(0) = \alpha_0; \tag{7}$$

conditions on the forward front of the perturbations s_N

$$\lambda(s_N) = \frac{2}{3}s_N^{3/2},\ \delta(s) = s_N^{-7/2},\ \alpha(s_N) = f(s_N) = \varphi(s_N) = 0; \tag{8}$$

and, conservation of energy

$$4\pi \int\limits_0^{s_N} \left(\frac{f}{\gamma - 1} + \frac{\alpha^2}{2}\right) ds = e_0. \tag{9}$$

Here the values of the dimensionless constants are as follows:

$$\alpha_0 = v_0 R^{-21/32}\rho_0^{-1/24}a^{3/16},\ e_0 = E_0 R^{-3/4}\rho_0^{-1/3}a^{1/2}. \tag{10}$$

The system (6) has the following first integrals

$$\begin{gathered} \alpha(s) = 0.75\lambda(s) - s/(2\delta(s)\lambda^2(s)). \\ 0.5s(f/(\gamma - 1) + \alpha^2/2) - \lambda^2(\delta f\alpha + \varphi) = \text{const}. \end{gathered} \tag{11}$$

where, taking into account (8), the constant in the second integral equals zero.

The Hugoniot relations [3] on an isothermal shock wave are:

$$\delta_2 = \delta_1/\psi,\ \psi = (2\lambda_1^2\delta_1/s_1)^2 f_1,\ f_2 = f_1,$$
$$\alpha_2 = \alpha_1 + (1-\psi)\frac{0.5s_1}{\delta_1\lambda_1^2},\ \varphi_2 = \varphi_1 + \frac{s_1}{4\lambda_1^2}\left(\frac{0.5s_1}{\delta_1\lambda_1^2}\right)^2(\psi^2-1) \tag{12}$$

(the indices 1 and 2 denote the values of the function in front of and behind the shock wave, respectively).

It can be shown analogously to [1, 2, 5] that in this problem the velocity of the forward front of the perturbations is finite, i.e., under the conditions (8) we have $s_N < \infty$, while between the piston and the forward front of the perturbations there should be an isothermal strong discontinuity, i.e., there exists a point $s_1 \in (0, s_N)$, where the conditions (12) are satisfied.

The values of s_N and s_1 are unique characteristic values of the boundary-value problem – there is a one-to-one correspondence between each admissible pair of quantities e_0, α_0 and the pair of quantities s_N, s_1.

In a neighborhood of the forward front of the perturbations s_N the solution has the form ($s \le s_N$)

$$\lambda(s) = \lambda_N + \frac{3\lambda_N}{2s_N}(s_N - s) + \ldots,\ \alpha(s) = \frac{2\lambda_N^2\delta_N}{s_N}a_1(s_N-s)^{2/5} + \ldots,$$
$$\delta(s) = \delta_N + \frac{4\delta_N^3\lambda_N^4}{s_N^2}a_1(s_N-s)^{2/5} + \ldots,\ f(s) = a_1(s_N-s)^{2/5} + \ldots, \tag{13}$$
$$\varphi(s) = \frac{3}{4}\frac{s_N}{\lambda_N^2}a_1(s_N-s)^{2/5} + \ldots,\ a_1 = \left(\frac{15}{8}\frac{s_N}{\delta_N\lambda_N^2}\right)^{2/5}.$$

If the positions of the singular points s_N and s_1 in the solution sought are known beforehand, then this solution can be found by numerically integrating the system (6) from the point $s = s_N - \tau$, at which the values of the functions are calculated based on the formulas (13), up to the piston $s = 0$ using at the point of discontinuity s_1 the relations (12).

The numerical experiments showed that for each value of the energy of the explosion e_0 there exists a critical value of the piston velocity $\alpha_0^*(e_0)$ such that in problems with $0 \le \alpha_0 \le \alpha_0^*(e_0)$, when integrating in the direction of the piston, there appears in a neighborhood of $s = 0$ an instability in the numerical solution, which is most characteristic for the behavior of the velocity $\alpha(s)$. This instability is of the same nature as the one in the numerical solution in solutions of the TW II type† for the piston problem [5], as well as in solutions of the problem of a pure explosion [2]. By the way, in [2] the behavior of integral curves was studied and iteration methods were given for solving the problem of a pure explosion, making it possible to avoid the instability in the numerical solution.

For brevity, in the rest of the presentation, we shall call the problem of the pure explosion ($\alpha_0 = 0$) problem A, the problem with maximum heat removal ($\alpha_0 \ne 0$, $f(0) = 0$) problem C, and the intermediate case ($\alpha_0 \ne 0$, $f(0) \ne 0$) problem B.

In the vicinity of the piston each of the mentioned problems has its own form of the power series expansion of the solution:

the problem A

$$\lambda(s) = \left(\frac{3}{\delta_0}\right)^{1/3}s^{1/3} + \ldots,\ \alpha(s) = \frac{7}{12}\left(\frac{3}{\delta_0}\right)^{1/3}s^{1/3} + \ldots,$$
$$\delta(s) = \delta_0 + \frac{1}{f_0}\left(\frac{35}{96} - \frac{\delta_0}{2f_0^{3/2}}\right)\left(\frac{\delta_0}{3}\right)^{1/3}s^{2/3} + \ldots, \tag{14}$$
$$f(s) = f_0 + \frac{1}{2f_0^{3/2}}\left(\frac{\delta_0}{3}\right)^{1/3}s^{2/3} + \ldots,$$
$$\varphi(s) = -\frac{2}{3}f_0\left(\frac{3}{2}\right)^{1/3}s^{1/3} + \ldots;$$

the problem B

$$\lambda(s) = \lambda_0 + \frac{1}{\delta_0\lambda_0^2}s + \ldots,$$
$$\alpha(s) = \frac{3}{4}\lambda_0 + \frac{1}{4\delta_0\lambda_0^2}s + \ldots,\ \delta(s) = \delta_0 + \frac{3}{4}\frac{1}{\lambda_0 f_0}\left(\frac{1}{4} - \frac{\delta_0}{f_0^{3/2}}\right)s + \ldots, \tag{15}$$
$$f(s) = f_0 + \frac{3}{4\lambda_0 f_0^{3/2}}s + \ldots,\ \varphi(s) = -\frac{3}{4}\lambda_0\delta_0 f_0 + \ldots;$$

† TW II – temperature wave of the second kind [5].

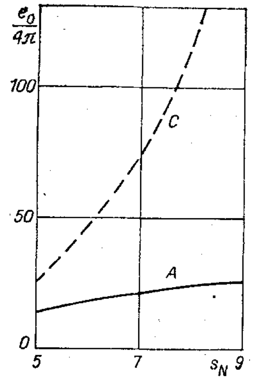

Fig. 3

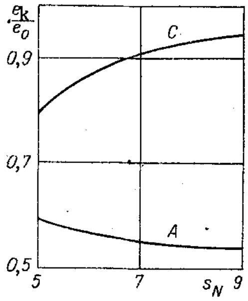

Fig. 4

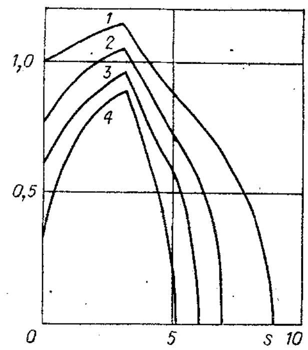

Fig. 5

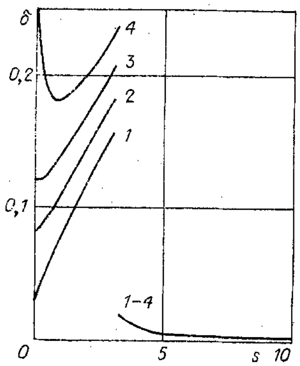

Fig. 6

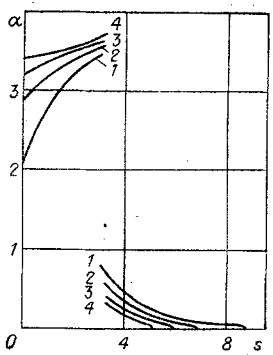

Fig. 7

the problem C

$$\begin{gathered}\lambda(s)=\lambda_0+\frac{5}{7}\frac{1}{\lambda_0^2\delta_1}s^{7/5}+\ldots,\\ \alpha(s)=\frac{3}{4}\lambda_0+\frac{1}{28}\frac{1}{\lambda_0^2\delta_1}s^{7/5}+\ldots,\\ \delta(s)=\delta_1 s^{-2/5}+\ldots,\ f(s)=f_1 s^{2/5}+\ldots,\\ \varphi(s)=-\frac{2}{5}\lambda_0^2\delta_1 f_1^{7/2}+\ldots\end{gathered}\tag{16}$$

Here δ_0, f_0 and δ_1, f_1 are unknown parameters.

The expansions (14)-(16) are employed in the course of the iteration process [2] in finding the solution in the problems $0 \le \alpha_0 \le \alpha_0^*(e_0)$.

Figure 1 shows the graphs of the position of the point of the discontinuity s_1 as a function of the position s_N in solutions of problems A and C. The points of the discontinuity in the solutions B fall between the curves A and C, since for fixed s_N the maximum value of s_1 is reached on the solution of the problem C, while the minimum value is reached in the problem A. This is shown in Fig. 2, which gives the values of the gas-dynamic and thermal functions on the piston as a function of s_1 in the solutions with the same position of the forward front of the perturbations $s_N = 7$. The figure also shows the dependence of the piston s_N of the values of the total and kinetic energy in the indicated solutions.

The dependence on s_1 shown in Fig. 2 is preserved for other values of s_N also. This is indicated by the graphs of the change in the total energy (Fig. 3) in the solutions of problems A (solid line) and C (broken line) as a function of s_N. In solutions of the problems A and C the characteristics (Fig. 4) of the changes of the relative fraction of the kinetic energy e_k/e_0 are interesting: in the solutions of the problem C it increases monotonically as s_N increases and therefore as the energy of the explosion e_0 increases, while in the solutions of problems A it decreases.

Figures 5-7 show the solutions of the problem B of the instantaneous energy release $e_0 = 4\pi 25.5$ with simultaneous motion of the heat-removing piston with velocities $\alpha_0 = 1.88$, 2.90, 3.22, and 3.40 (lines 1-4).

As we can see, as the velocity of the piston and, correspondingly, the heat removal on it increase, the velocity of the forward front of the perturbations decreases, and the velocity of the shock wave increases, and therefore the region of heating in front of the shock wave decreases in size. At the same time, the relative change in the velocity of the shock wave is small, while the velocity of the forward front of the perturbation wave is quite substantial.

The closeness of the values of the pressures (the graphs are not shown) directly behind the shock waves deserves special attention.

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MICROMECHANICS OF DYNAMIC DEFORMATION AND FAILURE

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As demonstrated in [1-4], dynamic deformation and failure of materials proceeds under conditions of marked distribution for particle velocity. This distribution governs not only the dependence of mechanical properties on deformation rate, but also material spalling resistance. The statistical nature of the occurrence of dynamic deformation and failure processes at the microlevel makes it possible by analogy with liquid and gas mechanics to use as a characteristic of these processes a distribution function for particle velocity which gives complete information about processes at the microlevel. However, for many practical purposes, it is entirely satisfactory to know only the first two features, i.e., average particle velocity and particle velocity dispersion. As will be shown below, these two characteristics may be determined simultaneously during a single act of shock loading for a specimen.

A study of material ductility and strength is often carried out on the basis of analyzing time profiles for loading and unloading waves, a record of which is accomplished by means of various types of fast-acting sensors, i.e., manganin, piezoceramic, variable capacitance etc. Laser interferometers occupy a special place among this type of recorder, making it possible to measure local dynamic movements and the free surface velocity of specimens. One of the main virtues of interferometers is their sensitivity to particle velocity distribution. Use of laser interferometry makes it possible not only to record the time profile of a shock wave, but also to obtain quantitative information about the evolution of the particle velocity distribution function at loading and unloading fronts. This information is particularly valuable in combination with microstructural studies of materials, since it makes it possible to study structural changes in the material during dynamic deformation. Whereas the time profile for the shock wave characterizes dynamic deformation and failure processes at the microlevel, the velocity distribution function and its features are microscopic characteristics of these processes.

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